This introductory graduate textbook provides a concise but accessible introduction to the Standard Model of particle physics. Throughout the book, theoretical concepts are developed clearly and carefully – from the electromagnetic and weak interactions of leptons and quarks to the strong interactions of quarks.

Chapters developing the theory are interspersed with chapters describing some of the wealth of experimental data supporting the model. To consolidate understanding, each chapter is rounded off with a set of problems and outline solutions. The book assumes only the standard mathematics taught in an undergraduate physics course; more sophisticated mathematical ideas are developed in the text and in appendices.

For graduate students in particle physics and physicists working in other fields who are interested in the current understanding of the ultimate constituents of matter, this textbook provides a lucid and up-to-date introduction.

Noel Cottingham and Derek Greenwood are theoreticians working in the H. H. Wills Physics Laboratory at the University of Bristol. Noel Cottingham is also a visiting professor at the Université Pierre et Marie Curie, Paris.

This book is their third collaboration: they have previously published two successful undergraduate texts, *An Introduction to Nuclear Physics* and *Electricity and Magnetism*, both for Cambridge University Press.

An Introduction to the Standard Model of Particle Physics

An Introduction to the Standard Model of Particle Physics

W. N. COTTINGHAM and D. A. GREENWOOD

CAMBRIDGE
UNIVERSITY PRESS

PUBLISHED BY THE PRESS SYNDICATE OF THE UNIVERSITY OF CAMBRIDGE
The Pitt Building, Trumpington Street, Cambridge CB2 1RP, United Kingdom

CAMBRIDGE UNIVERSITY PRESS
The Edinburgh Building, Cambridge CB2 2RU, UK http://www.cup.cam.ac.uk
40 West 20th Street, New York, NY 10011-4211, USA http://www.cup.org
10 Stamford Road, Oakleigh, Melbourne 3166, Australia

First published 1998

Printed in the United Kingdom at the University Press, Cambridge

Typeset in Times 10/13pt [VN]

A catalogue record for this book is available from the British Library

Library of Congress Cataloguing in Publication data

Cottingham, W. N.
An introduction to the standard model of particle physics / W.N. Cottingham and D.A. Greenwood
p. cm.
Includes index.
ISBN 0 521 58191 5 (hb)
1. Standard model (Nuclear physics) I. Greenwood, D. A.
II. Title.
QC794.6.S75C68 1998
539.7′2—dc21 98–14144 CIP

ISBN 0 521 58191 5 hardback
ISBN 0 521 58832 4 paperback

Contents

Preface

The 'Standard Model' of particle physics is the result of an immense experimental and inspired theoretical effort, spanning more than fifty years. This book is intended as a concise but accessible introduction to the elegant theoretical edifice of the Standard Model. With the planned construction of the Large Hadron Collider at CERN now agreed, the Standard Model will continue to be a vital and active subject.

The beauty and basic simplicity of the theory can be appreciated at a certain 'classical' level, treating the boson fields as true classical fields and the fermion fields as completely anticommuting. To make contact with experiment the theory must be quantised. Many of the calculations of the consequences of the theory are made in quantum perturbation theory. Those we present are for the most part to the lowest order of perturbation theory only, and do not have to be renormalised. Our account of renormalisation in Chapter 8 is descriptive, as is also our final Chapter 19 on the anomalies that are generated upon quantisation.

A full appreciation of the success and significance of the Standard Model requires an intimate knowledge of particle physics that goes far beyond what is usually taught in undergraduate courses, and cannot be conveyed in a short introduction. However, we attempt to give an overview of the intellectual achievement represented by the Model, and something of the excitement of its successes. In Chapter 1 we give a brief résumé of the physics of particles as it is qualitatively understood today. Later chapters developing the theory are interspersed with chapters on the experimental data. The amount of supporting data is immense and so we attempt to focus only on the most salient experimental results. Unless otherwise referenced, experimental values quoted are those recommended by the Particle Data Group (1996).

The mathematical background assumed is that usually acquired during an undergraduate physics course. In particular, a facility with the manipulations of matrix algebra is very necessary; Appendix A provides an *aide-mémoire*. Principles of symmetry play an important rôle in the construction of the model, and Appendix B is a self-contained account of the group theoretic ideas we use in describing these symmetries. The mathematics we require is not technically difficult, but the reader must accept a gradually more abstract formulation of physical theory than that presented at undergraduate level. Detailed derivations which would impair the flow of the text are often set as problems (and outline solutions to these are provided).

The book is based on lectures given to beginning graduate students at the University of Bristol, and is intended for use at this level, and perhaps, in part at least, at senior undergraduate level. It is not intended only for the dedicated particle physicist: we hope it may be read by physicists working in other fields who are interested in the present understanding of the ultimate constituents of matter.

We should like to thank the anonymous referees of Cambridge University Press for their useful comments on our proposals. The Department of Physics at Bristol has been generous in its encouragement of our work. Many colleagues, at Bristol and elsewhere, have contributed to our understanding of the subject. We are grateful to Mrs Victoria Parry for her careful and accurate work on the typescript, without which this book would never have appeared.

W. N. Cottingham
D. A. Greenwood

Notation

Position vectors in three-dimensional space are denoted by $\mathbf{r} = (x,y,z)$, or $\mathbf{x} = (x^1,x^2,x^3)$ where $x^1 = x$, $x^2 = y$, $x^3 = z$.

A general vector $\mathbf{a}$ has components (a^1,a^2,a^3), and $\hat{\mathbf{a}}$ denotes a unit vector in the direction of $\mathbf{a}$.

Volume elements in three-dimensional space are denoted by $\mathrm{d}^3\mathbf{x} = \mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z = \mathrm{d}x^1\mathrm{d}x^2\mathrm{d}x^3$.

The coordinates of an event in four-dimensional time and space are denoted by $x = (x^0,x^1,x^2,x^3) = (x^0,\mathbf{x})$ where $x^0 = ct$.

Volume elements in four-dimensional time and space are denoted by $\mathrm{d}^4x = \mathrm{d}x^0\mathrm{d}x^1\mathrm{d}x^2\mathrm{d}x^3 = c\,\mathrm{d}t\,\mathrm{d}^3\mathbf{x}$.

Greek indices μ,ν,λ,ρ take on the values 0,1,2,3.

Latin indices i,j,k,l take on the space values 1,2,3.

Pauli matrices

We denote by σ^μ the set $(\sigma^0,\sigma^1,\sigma^2,\sigma^3)$ and by $\tilde{\sigma}^\mu$ the set $(\sigma^0,-\sigma^1,-\sigma^2,-\sigma^3)$, where

$$\sigma^0 = \mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma^2 = \begin{pmatrix} 0 & -\mathrm{i} \\ \mathrm{i} & 0 \end{pmatrix}, \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},$$

$$(\sigma^1)^2 = (\sigma^2)^2 = (\sigma^3)^2 = \mathbf{I}; \qquad \sigma^1\sigma^2 = \mathrm{i}\sigma^3 = -\sigma^2\sigma^1, \text{ etc.}$$

Chiral representation for γ-matrices

$$\gamma^0 = \begin{pmatrix} \mathbf{0} & \mathbf{I} \\ \mathbf{I} & \mathbf{0} \end{pmatrix}, \qquad \gamma^i = \begin{pmatrix} \mathbf{0} & \sigma^i \\ -\sigma^i & \mathbf{0} \end{pmatrix},$$

$$\gamma^5 = \mathrm{i}\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} -\mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{pmatrix}.$$

Quantisation ($\hbar = c = 1$)

$(E,\mathbf{p}) \to (\mathrm{i}\partial/\partial t, -\mathrm{i}\nabla)$, or $p^\mu \to \mathrm{i}\partial^\mu$.

For a particle carrying charge q in an external electromagnetic field,

$(E,\mathbf{p}) \to (E - q\phi, \mathbf{p} - q\mathbf{A})$, or $p^\mu \to p^\mu - qA^\mu$,
$\mathrm{i}\partial^\mu \to (\mathrm{i}\partial^\mu - qA^\mu) = \mathrm{i}(\partial^\mu + \mathrm{i}qA^\mu)$.

Field definitions

$$Z_\mu = W_\mu{}^3 \cos\theta_\mathrm{w} - B_\mu \sin\theta_\mathrm{w},$$
$$A_\mu = W_\mu{}^3 \sin\theta_\mathrm{w} + B_\mu \cos\theta_\mathrm{w},$$

where $\sin^2\theta_\mathrm{w} = 0.2315(4)$

$$g_2 \sin\theta_\mathrm{w} = g_1 \cos\theta_\mathrm{w} = e. \qquad G_\mathrm{F} = g_2{}^2/(4\sqrt{2}\,M_\mathrm{w}{}^2).$$

Glossary of symbols

$\mathbf{A}$	electromagnetic vector potential Section 4.3
A^μ	electromagnetic four-vector potential
$A^{\mu\nu}$	field strength tensor Section 11.3
A_FB	forward–backward asymmetry Section 15.2
a	wave amplitude Section 3.5
$a,a^\dagger$	boson annihilation, creation operator
$\mathbf{B}$	magnetic field
B^μ	gauge field Section 11.1
$B^{\mu\nu}$	field strength tensor Section 11.2
$b,b^\dagger$	fermion annihilation, creation operator
$\mathbf{D}$	isospin doublet Section 16.6
$d,d^\dagger$	antifermion annihilation, creation operator
d_k	($k = 1,2,3$) down-type quark field
$\mathbf{E}$	electric field
E	energy
$e,e_\mathrm{L},e_\mathrm{R}$	electron Dirac, two-component left-handed, right-handed field
$F^{\mu\nu}$	electromagnetic field strength tensor Section 4.1
f	radiative corrections factor Sections 15.1, 17.4
f_{abc}	structure constants of *SU(3)* Section B.7
$\mathbf{G}^\mu$	gluon matrix gauge field
$\mathbf{G}^{\mu\nu}$	gluon field strength tensor
G_F	Fermi constant Section 9.4
$g_{\mu\nu}$	metric tensor
g	strong coupling constant Section 16.1
g_1,g_2	electroweak coupling constants

H	Hamiltonian Section 3.1
$h(x)$	Higgs field
$\mathscr{H}$	Hamiltonian density Section 3.3
$\mathbf{I}$	isospin operator Sections 1.5, 16.6
$\mathbf{J}$	electric current density Section 4.1
$\mathbf{J}$	total angular momentum operator
J	Jarlskog constant Section 14.3
J^μ	lepton number current Section 12.4
$\mathbf{j}$	probability current Section 7.1
j^μ	lepton current Section 12.2
K	string tension Section 17.1
$\mathbf{k}$	wave-vector
$\mathbf{L}$	lepton doublet Section 12.1
L	Lagrangian Section 3.1
$\mathscr{L}$	Lagrangian density Section 3.3
l^3	normalisation volume Section 3.5
$\mathbf{M}$	left-handed spinor transformation matrix Section B.6
M	proton mass Section D.1
m	mass
$\mathbf{N}$	right-handed spinor transformation matrix Section B.6
N	number operator Section C.1
$\hat{\mathbf{O}}$	quantum operator
$\mathbf{P}$	total field momentum
$\mathbf{p}$	momentum
Q^2	$= -q_\mu q^\mu$
$\mathbf{q}$	quark colour triplet
q^μ	energy–momentum transfer
$\mathbf{R}$	rotation matrix Section B.2
$\mathbf{S}$	spin operator
S	action Section 3.1
s	square of centre of mass energy
T^μ_ν	energy–momentum tensor Section 3.6
$\mathbf{U}$	unitary matrix
u_k	($k = 1,2,3$) up-type quark field
u_L, u_R	two-component left-handed, right-handed spinors Section 6.1
u_+, u_-	Dirac spinors Section 6.3
$\mathbf{V}$	Kobayashi–Maskawa matrix Section 14.2
V	normalisation volume
$\mathbf{v}$	velocity
v	$= \lvert \mathbf{v} \rvert$
v_L, v_R	two-component left-handed, right-handed spinors
v_+, v_-	Dirac spinors Section 6.4
$\mathbf{W}^\mu$	matrix of vector gauge field Section 11.1

$\mathbf{W}^{\mu\nu}$	field strength tensor Section 11.2
$W_\mu^1, W_\mu^2, W_\mu^+, W_\mu^-$	fields of W boson
Z_μ	field of Z boson
$\alpha(Q^2)$	effective fine structure constant Section 16.3
$\alpha_s(Q^2)$	effective strong coupling constant Section 16.3
α_{latt}	lattice coupling constant Section 17.1
α^i	Dirac matrix Section 5.1
β	Dirac matrix Section 5.1
β	$= v/c$
Γ	width of excited state, decay rate
γ^μ	Dirac matrix Section 5.5
γ	$= (1 - \beta^2)^{-1/2}$
δ	Kobayashi–Maskawa phase Section 14.3
$\boldsymbol{\varepsilon}$	polarisation unit vector Section 4.7
ε	helicity index
θ	boost parameter: $\tanh\theta = \beta$, $\cosh\theta = \gamma$ Section 2.1, phase angle, scattering angle, scalar potential Section 4.3, gauge parameter field Section 10.2
θ_w	Weinberg angle
Λ^{-1}	confinement length Section 16.3
Λ_{latt}	lattice parameter Section 17.1
λ_a	matrices associated with $SU(3)$ Section B.7
μ, μ_L, μ_R	muon Dirac, two-component left-handed, right-handed field
$\nu_{eL}, \nu_{\mu L}, \nu_{\tau L}$	electron neutrino, muon neutrino, tau neutrino field
Π	momentum density Section 3.3
ρ	electric charge density
$\rho(E)$	density of final states at energy E
Σ	spin operator acting on Dirac field Section 6.2
τ	mean life
τ, τ_L, τ_R	tau Dirac, two-component left-handed, right-handed field
Φ	complex scalar field Section 3.7
ϕ	real scalar field Section 2.3, scalar potential Section 4.1, gauge parameter field Section 10.2
ϕ_0	vacuum expectation value of the Higgs field
χ	gauge parameter field Section 4.3, scalar field Section 10.3
ψ	four-component Dirac field
ψ_L, ψ_R	two-component left-handed, right-handed spinor field
$\bar{\psi}$	$\psi^\dagger\gamma^0$ Section 5.5
ω	frequency

1 The particle physicist's view of Nature

1.1 Introduction

It is more than a century since the discovery by J. J. Thomson of the electron. The electron is still thought to be a structureless point particle, and one of the elementary particles of Nature. Other particles which were subsequently discovered and at first thought to be elementary, like the proton and the neutron, have since been found to have a complex structure.

What then are the ultimate constituents of matter? How are they categorised? How do they interact with each other? What, indeed, should we ask of a mathematical theory of elementary particles? Since the discovery of the electron, and more particularly in the last fifty years, there has been an immense amount of experimental and theoretical effort to determine answers to these questions. The present Standard Model of particle physics stems from that effort.

The Standard Model asserts that the material in the universe is made up of elementary fermions interacting through fields, of which they are the sources. The particles associated with the interaction fields are bosons.

Four types of interaction field, set out in Table 1.1, have been distinguished in Nature. On the scales of particle physics, gravitational forces are insignificant. The Standard Model excludes from consideration the gravitational field. The quanta of the electromagnetic interaction field between electrically charged fermions are the massless photons. The quanta of the weak interaction fields between fermions are the charged W^+ and W^- bosons and the neutral Z boson, discovered at CERN in 1983. Since these carry mass, the weak interaction is short ranged: by the uncertainty principle, a particle of mass M can exist as part of an intermediate state for a time $\hbar/Mc^2$, and in this time the particle can travel a distance no greater than $\hbar c/Mc^2$. Since $M_{\mathrm{w}} \approx 80\,\mathrm{GeV}/c^2$ and $M_{\mathrm{z}} \approx 90\,\mathrm{GeV}/c^2$, the weak interaction has a range $\approx 10^{-3}\,\mathrm{fm}$.

The quanta of the strong interaction field, the gluons, have zero mass, and, like

Table 1.1. *Types of interaction field*

Interaction field	Boson	Spin
Gravitational field	'Gravitons' postulated	2
Weak field	W^+, W^-, Z particles	1
Electromagnetic field	Photons	1
Strong field	'Gluons' postulated	1

photons, might be expected to have infinite range. However, unlike the electromagnetic field, the gluon fields are *confining*, a property we shall be discussing at length in the later chapters of this book.

The elementary fermions of the Standard Model are of two types: *leptons* and *quarks*. All have spin $\frac{1}{2}$, in units of $\hbar$, and in isolation would be described by the Dirac equation, which we discuss in Chapters 5, 6 and 7. Leptons interact only through the electromagnetic interaction (if they are charged) and the weak interaction. Quarks interact through the electromagnetic and weak interactions and also through the strong interaction.

1.2 The construction of the Standard Model

Any theory of elementary particles must be consistent with special relativity. The combination of quantum mechanics, electromagnetism and special relativity led Dirac to the equation now universally known as the *Dirac equation* and, on quantising the fields, to quantum field theory. Quantum field theory had as its first triumph quantum electrodynamics, QED for short, which describes the interaction of the electron with the electromagnetic field. The success of a post-war generation of physicists, Feynman, Schwinger, Tomonaga, Dyson and others, in handling the infinities which arise in the theory led to a spectacular agreement between QED and experiment, which we describe in Chapter 8.

The Standard Model, like the QED it contains, is a theory of interacting fields. Our emphasis will be on the beauty and simplicity of the theory, and this can be understood at a certain 'classical' level, treating the boson fields as true classical fields, and the fermion fields as completely anticommuting. To make a judgement of the success of the model in describing the data, it is necessary to quantise the fields, but to keep this book concise and accessible, results beyond the lowest orders of perturbation theory will only be quoted.

The construction of the Standard Model has been guided by principles of symmetry. The mathematics of symmetry is provided by group theory; groups of particular significance in the formulation of the Model are described in Appendix B. The connection between symmetries and physics is deep. *Noether's theorem* states, essentially, that for every continuous symmetry of Nature there is a corresponding conservation law. For example, it follows from the presumed homogeneity of space

Table 1.2. *Leptons*

	Mass (MeV/c^2)	Mean life (s)	Electric charge
Electron e^-	0.5110	∞	$-e$
Electron neutrino ν_e	$< 15 \times 10^{-6}$	∞?	0
Muon μ^-	105.658	2.197×10^{-6}	$-e$
Muon neutrino ν_μ	< 0.17	∞?	0
Tau τ^-	1777	$(291.0 \pm 1.5) \times 10^{-15}$	$-e$
Tau neutrino ν_τ	< 24	∞?	0

and time that the Lagrangian of a closed system is invariant under uniform translations of the system in space and in time. Such transformations are therefore symmetry operations on the system. It may be shown that they lead, respectively, to the laws of conservation of momentum and conservation of energy. Symmetries, and symmetry breaking, will play a large part in this book.

In the following sections of this chapter, we remind the reader of some of the salient discoveries of particle physics, which the Standard Model must incorporate. In Chapter 2 we begin on the mathematical formalism we shall need in the construction of the Standard Model.

1.3 Leptons

The known leptons are listed in Table 1.2. The Dirac equation for a charged massive fermion predicts, correctly, the existence of an *antiparticle* of the same mass and spin, but opposite charge, and opposite magnetic moment relative to the direction of the spin. The Dirac equation for a massless neutrino ν predicts the existence of an antineutrino $\bar{\nu}$.

Of the charged leptons, only the electron e^- carrying charge $-e$ and its antiparticle e^+, are stable. The muon μ^- and tau τ^- and their antiparticles, the μ^+ and τ^+, differ from the electron and positron only in their masses and their finite lifetimes. They appear to be elementary particles. In this book we take the neutrinos to have zero mass. The experimental situation regarding small neutrino masses has not yet been clarified. Non-zero neutrino masses would have dramatic implications for cosmology. There is good experimental evidence that the e, μ and τ have different neutrinos ν_e, ν_μ and ν_τ associated with them.

It is believed to be true of all interactions that they preserve electric charge. Leaving aside the possibility of neutrino masses, it seems that in its interactions a lepton can change only to another of the same type, and a lepton and an antilepton of the same type can only be created or destroyed together. These laws are exemplified in the decay

$$\mu^- \to \nu_\mu + e^- + \bar{\nu}_e.$$

Table 1.3. *Properties of quarks*

Quark	Electric charge (e)	Mass
Up u	2/3	2–8 MeV
Down d	−1/3	5–15 MeV
Charmed c	2/3	1–1.6 GeV
Strange s	−1/3	100–300 MeV
Top t	2/3	168–192 GeV
Bottom b	−1/3	4.1–4.5 GeV

This *conservation of lepton number*, antileptons being counted negatively, which holds for each separate type of lepton, along with the conservation of electric charge, will be apparent (aside from the considerations of Chapter 19) in the Standard Model.

1.4 Quarks and systems of quarks

The known quarks are listed in Table 1.3. In the Standard Model, quarks, like leptons, are spin $\frac{1}{2}$ Dirac fermions, but the electric charges they carry are 2e/3, −e/3. Quarks carry quark number, antiquarks being counted negatively. The net quark number of an isolated system has never been observed to change. However, the number of different types or *flavours* of quark are not separately conserved: changes are possible through the weak interaction.

A difficulty with the experimental investigation of quarks is that an isolated quark has never been observed. Quarks are always confined in compound systems which extend over distances of about 1 fm. The most elementary quark systems are *baryons* which have net quark number three, and *mesons* which have net quark number zero. In particular, the proton and neutron are baryons. Mesons are essentially a quark and an antiquark, bound transiently by the strong interaction field. The term *hadron* is used generically for a quark system.

The proton basically contains two *up* quarks and one *down* quark (uud), and the neutron two down quarks and one up (udd). The proton is the only stable baryon. The neutron is a little more massive than the proton, by about 1.3 MeV/c^2, and in free space it decays to a proton through the weak interaction: $\mathrm{n} \to \mathrm{p} + \mathrm{e}^- + \bar{\nu}_\mathrm{e}$, with a mean life of about 15 minutes.

All mesons are unstable. The lightest mesons are the π-mesons or 'pions'. The electrically charged π^+ and π^- are made up of $(\mathrm{u}\bar{\mathrm{d}})$ and $(\bar{\mathrm{u}}\mathrm{d})$ pairs, respectively, and the neutral π^0 is either $\mathrm{u}\bar{\mathrm{u}}$ or $\mathrm{d}\bar{\mathrm{d}}$, with equal probabilities; it is a coherent superposition $(\mathrm{u}\bar{\mathrm{u}} - \mathrm{d}\bar{\mathrm{d}})/\sqrt{2}$ of the two states. The π^+ and π^- have a mass of 139.57 MeV/c^2 and the π^0 is a little lighter, 134.98 MeV/c^2. The next lightest meson is the $\eta(\approx 547\,\mathrm{MeV}/c^2)$, which is the combination $(\mathrm{u}\bar{\mathrm{u}} + \mathrm{d}\bar{\mathrm{d}})/\sqrt{2}$ of quark–antiquark pairs orthogonal to the π^0, with some $\mathrm{s}\bar{\mathrm{s}}$ component.

1.5 Spectroscopy of systems of light quarks

As will be discussed in Chapter 16, the masses of the u and d quarks are quite small, of the order of a few MeV/c^2, closer to the electron mass than to a meson or baryon mass. A u or d quark confined within a distance $\approx 1\,\mathrm{fm}$ has, by the uncertainty principle, a momentum $p \approx \hbar/(1\,\mathrm{fm}) \approx 200\,\mathrm{MeV}/c$, and hence its energy is $E \approx pc \approx 200\,\mathrm{MeV}$, almost independent of the quark mass. All quarks have the same strong interactions. As a consequence the physics of light quark systems is almost independent of the quark masses. There is an approximate $SU(2)$ isospin symmetry (Section 16.6), which is evident in the Standard Model.

The symmetry is not exact because of the different quark masses and different quark charges. The symmetry breaking due to quark mass differences prevails over the electromagnetic. In all cases where two particles differ only in that a d quark is substituted for a u quark, the particle with the d quark is more massive. For example, the neutron is more massive than the proton, even though the mass, $\sim 2\,\mathrm{MeV}/c^2$, associated with the electrical energy of the charged proton is far greater than that associated with the (overall neutral) charge distribution of the neutron. We conclude that the d quark is heavier than the u quark.

The evidence for the existence of quarks came first from nucleon spectroscopy. The proton and neutron have many excited states which appear as resonances in photon–nucleon scattering and in pion–nucleon scattering (Fig. 1.1). Hadron states containing light quarks can be classified using the concept of isospin. The u and d quarks are regarded as a doublet of states $|\mathrm{u}\rangle$ and $|\mathrm{d}\rangle$, with $I = 1/2$ and $I_3 = +1/2$, $-1/2$, respectively. The total isospin of a baryon made up of three u or d quarks is then $I = 3/2$ or $I = 1/2$. The isospin 3/2 states make up multiplets of four states almost degenerate in energy but having charges $2e$(uuu), e(uud), 0(udd), $-e$(ddd). The $I = 1/2$ states make up doublets, like the proton and neutron, having charges e(uud) and 0(udd). The electric charge assignments of the quarks were made to comprehend this baryon charge structure.

Energy level diagrams of the $I = 3/2$ and $I = 1/2$ states up to excitation energies of 1 GeV are shown in Fig. 1.2. The energy differences between states in a multiplet are only of the order of 1 MeV and cannot be shown on the scale of the figure. The widths Γ of the excited states are however quite large, of the order of 100 MeV, corresponding to mean lives $\tau = \hbar/\Gamma \sim 10^{-23}\,\mathrm{s}$. The excited states are all energetic enough to decay through the strong interaction, as for example $\Delta^{++} \to \mathrm{p} + \pi^+$ (Fig. 1.3).

The rich spectrum of the baryon states can largely be described and understood on the basis of a simple 'shell' model of three confined quarks. The lowest states have orbital angular momentum $L = 0$ and positive parity. The states in the next group have $L = 1$ and negative parity, and so on. However, the model has the curious feature that, to fit the data, the states are completely symmetric in the interchange of any two quarks. For example, the Δ^{++}(uuu), which belongs to the lowest $I = 3/2$ multiplet, has $J^{\mathrm{P}} = 3/2^+$. If $L = 0$ the three quark spins must be aligned $\uparrow\uparrow\uparrow$ in a symmetric state to give $J = 3/2$, and the lowest energy spatial state

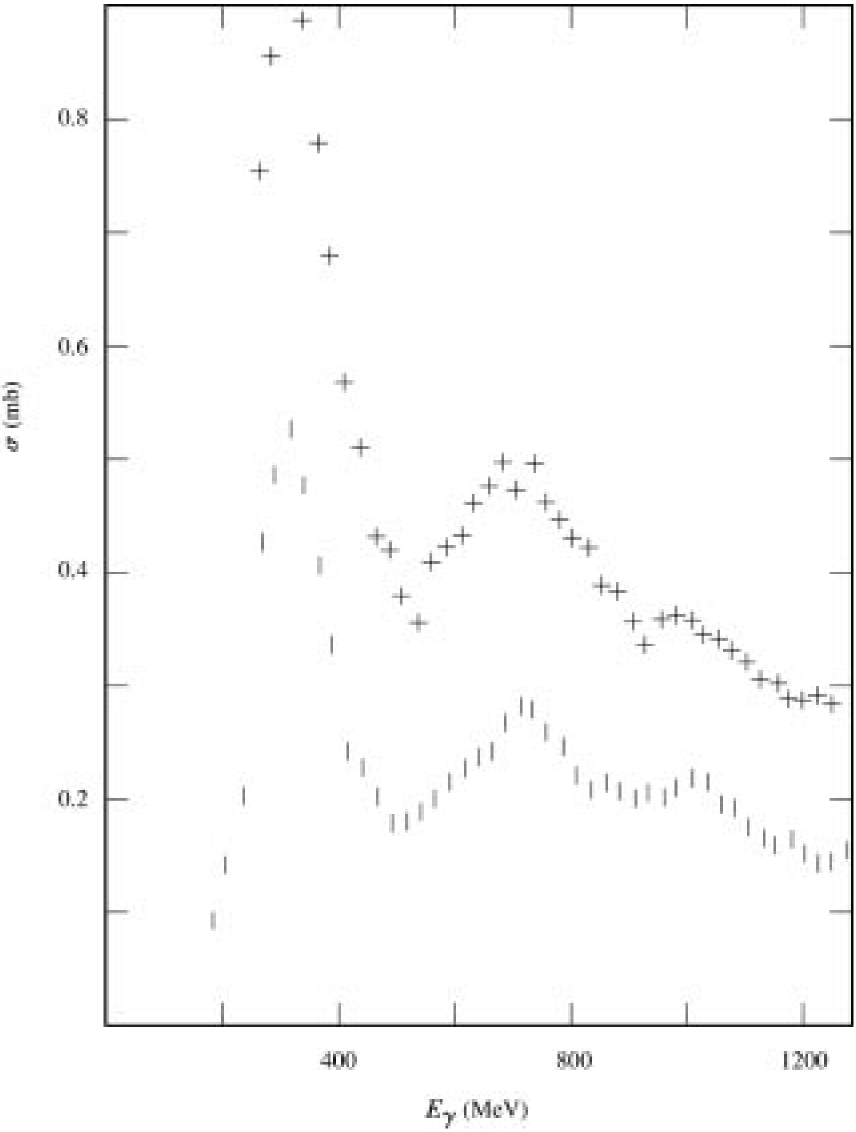

Figure 1.1 The photon cross-section for hadron production by photons on protons (dashes) and deuterons (crosses). The difference between these cross-sections is approximately the cross-section for hadron production by photons on neutrons. (After Armstrong *et al.* (1972).)

must be totally symmetric. Symmetry under interchange is not allowed for an assembly of identical fermions! However, there is no doubt that the model demands symmetry, and with symmetry it works very well. The resolution of this problem will be left to later in this chapter. There are only a few states (broken lines in Fig. 1.2) which cannot be understood within the simple shell model.

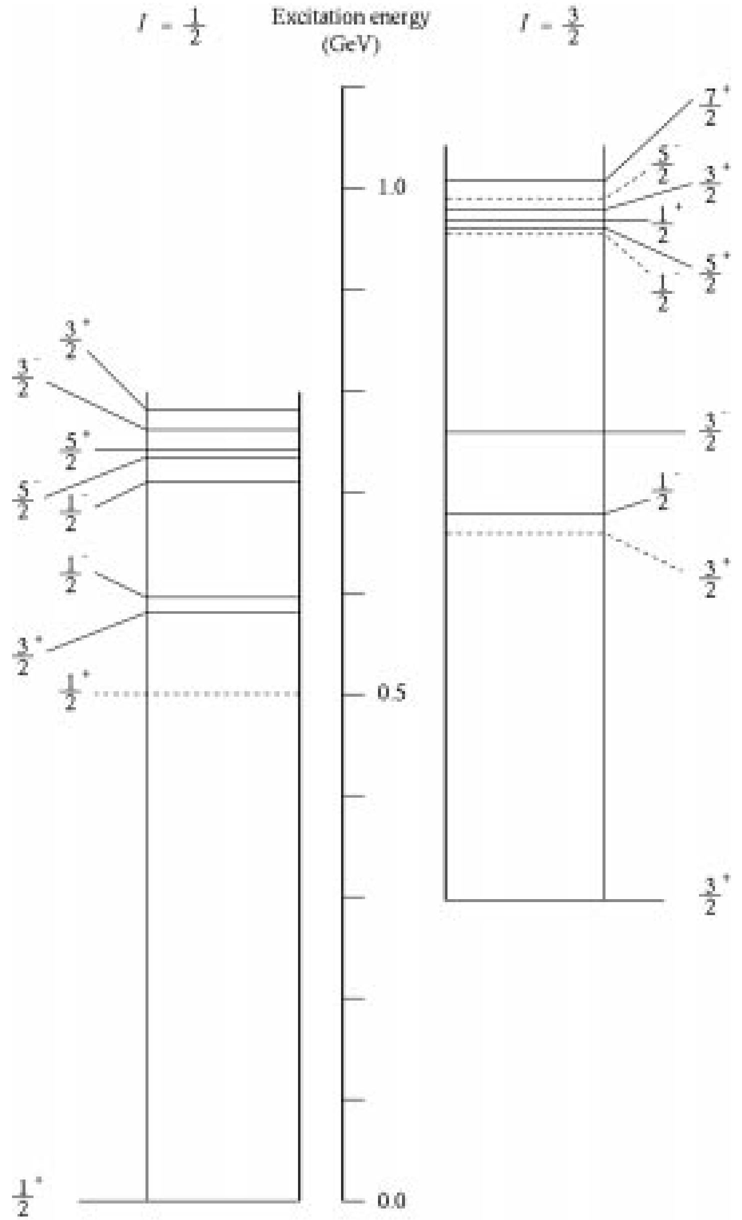

Figure 1.2 An energy-level diagram for the nucleon and its excited states. The levels fall into two classes, isotopic doublets ($I = 1/2$) and isotopic quartets ($I = 3/2$). The states are labelled by their total angular momenta and parities J^P. The nucleon doublet N(939) is the ground state of the system, the Δ (1232) is the lowest lying quartet. Within the quark model (see text) these two states are the lowest that can be formed with no quark orbital angular momentum ($L = 0$). The other states designated by unbroken lines have clear interpretations: they are all the next most simple states with $L = 1$ (negative parity) and $L = 2$ (positive parity). The broken lines show states that have no clear interpretation within the simple three-quark model. They are perhaps associated with excited states of the gluon fields.

Table 1.4. *Isospin quantum numbers of light quarks*

Quark	Isospin I	I_3
u	1/2	1/2
$\bar{u}$	1/2	$-1/2$
d	1/2	$-1/2$
$\bar{d}$	1/2	1/2
s	0	0
$\bar{s}$	0	0

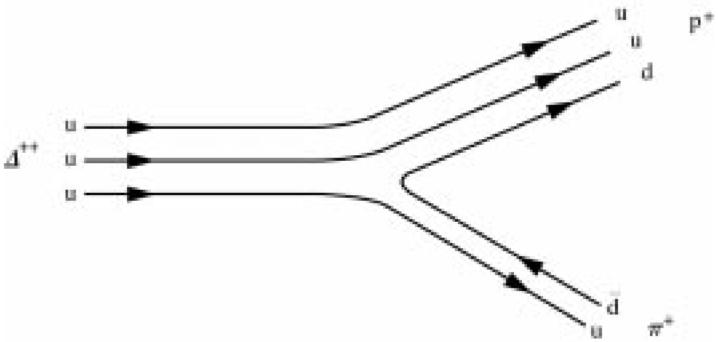

Figure 1.3 A quark model diagram of the decay $\Delta^{++} \rightarrow p + \pi^{+}$. The gluon field is not represented in this diagram, but it would be responsible for holding the quark systems together and for the creation of the $d\bar{d}$ pair.

Mesons made up of light u and d quarks and their antiquarks also have a rich spectrum of states which can be classified by their isospin. Antiquarks have an I_3 of opposite sign to that of their corresponding quark (Table 1.4). By the rules for the addition of isospin, quark–antiquark pairs have $I = 0$ or $I = 1$. The $I = 0$ states are singlets with charge 0, like the η (Fig. 1.4(a)). The $I = 1$ states make up triplets carrying charge $+e, 0, -e$, which are almost degenerate in energy, like the triplet $\pi^{+}, \pi^{0}, \pi^{-}$.

The spectrum of $I = 1$ states with energies up to 1.5 GeV is shown in Fig. 1.4(b). As in the baryon case the splitting between states in the same isotopic multiplet is only a few MeV; the widths of the excited states are like the widths of the excited baryon states, of the order of 100 MeV. In the lowest multiplet (the pions), the quark–antiquark pair is in an $L = 0$ state with spins coupled to zero. Hence $J^{P} = 0^{-}$, since a fermion and antifermion have opposite relative parity (Section 6.4). In the first excited state the spins are coupled to 1 and $J^{P} = 1^{-}$. These are the ρ mesons. With $L = 1$ and spins coupled to $S = 1$ one can construct states $2^{+}, 1^{+}, 0^{+}$, and with $L = 1$ and spins coupled to $S = 0$ a state 1^{+}. All these states can be identified in Fig. 1.4(b).

1.6 More quarks

'Strange' mesons and baryons were discovered in the late 1940s, soon after the discovery of the pions. It is apparent that as well as the u and d quarks there exists a so-called *strange* quark s, and strange particles contain one or more s quarks. An s quark can replace a u or d quark in any baryon or meson to make the strange baryons and strange mesons. The electric charges show that the s quark, like the d, has charge $-e/3$, and the spectra can be understood if the s is assigned isospin $I = 0$.

The lowest mass strange mesons are the $I = 1/2$ doublet, K^- ($s\bar{u}$, mass 494 MeV) and K^0 ($d\bar{s}$, mass 498 MeV). Their antiparticles make up another doublet, the $K^+(u\bar{s})$ and $\bar{K}^0(\bar{d}s)$.

The effect of quark replacement on the meson spectrum is illustrated in Fig. 1.4. Each level in the spectrum of Fig. 1.4(b) has a member ($d\bar{u}$) with charge $-e$. Fig. 1.4(c) shows the spectrum of strange ($s\bar{u}$) mesons. There is a correspondence in angular momentum and parity between states in the two spectra. The energy differences are a consequence of the s quark having a much larger mass, of the order of 200 MeV.

The excess of mass of the s quark over the u and d quarks makes the s quark in any strange particle unstable to decay by the weak interaction.

Besides the u, d and s quarks there are considerably heavier quarks: the *charmed* quark c (mass $\approx 1.3\,\mathrm{GeV}/c^2$, charge $2e/3$), the *bottom* quark b (mass $\approx 4.3\,\mathrm{GeV}/c^2$, charge $-e/3$), and the *top* quark t (mass $\approx 180\,\mathrm{GeV}/c^2$, charge $2e/3$). The quark masses are most remarkable, being even more disparate than the lepton masses. The experimental investigation of the elusive top quark is still in its infancy, but it seems that three quarks of any of the six known flavours can be bound to form a system of states of a baryon (or three antiquarks to form antibaryon states), and any quark–antiquark pair can bind into mesonic states.

The c and b quarks were discovered in e^+e^- colliding beam machines. Very prominent narrow resonances were observed in the e^+e^- annihilation cross-sections. Their widths, of less than 15 MeV, distinguished the meson states responsible from those made up of u, d or s quarks. There are two groups of resonant states. The group at around 3 GeV centre of mass energy are known as J/Ψ resonances, and are interpreted as *charmonium* $c\bar{c}$ states. Another group around 10 GeV, the Υ (upsilon) resonances, are interpreted as *bottomonium* $b\bar{b}$ states. The current state of knowledge of the $c\bar{c}$ and $b\bar{b}$ energy levels is displayed in Fig. 1.5. We shall discuss these systems in Chapter 17.

The existence of the top quark was established in 1995 at Fermilab, in $\bar{p}p$ collisions.

1.7 Quark colour

Much informative quark physics has been revealed in experiments with e^+e^- colliding beams. We mention here experiments in the range between centre of mass

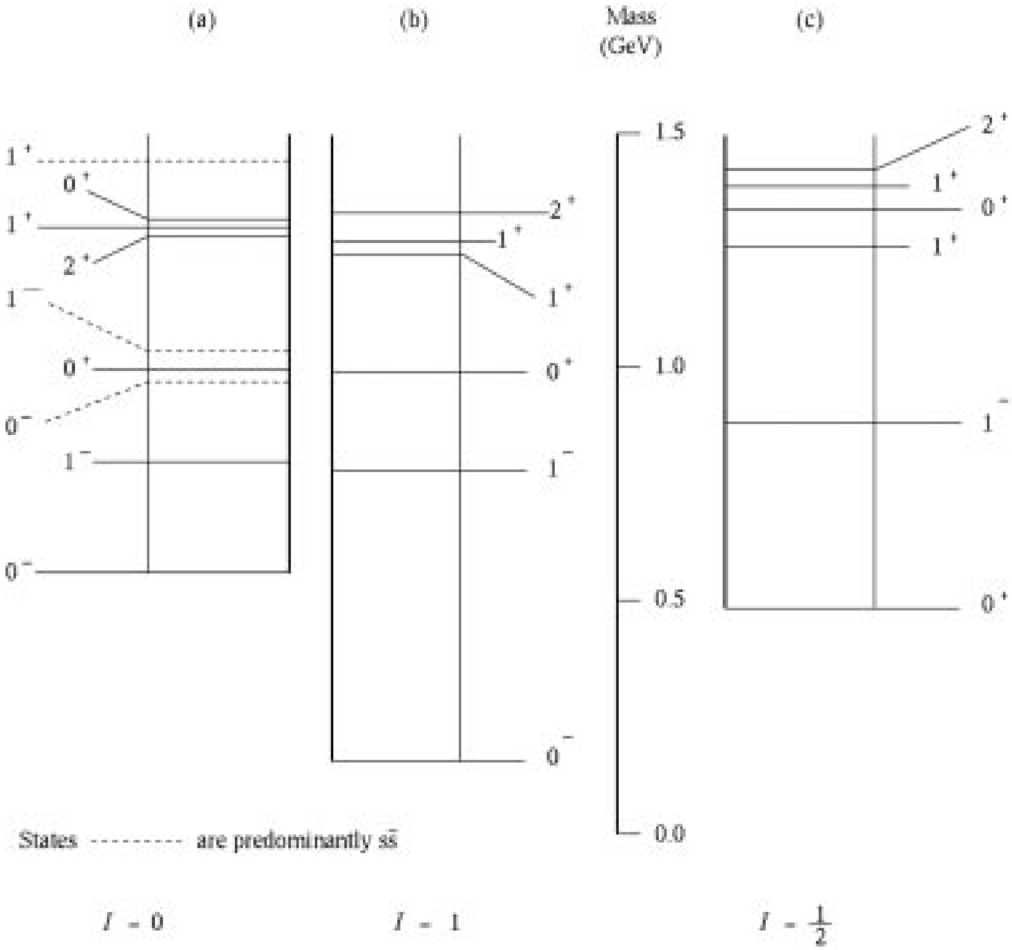

Figure 1.4 States of the quark–antiquark system $u\bar{u}$, $u\bar{d}$, $d\bar{u}$, $d\bar{d}$ form isotopic triplets ($I = 1$): $u\bar{d}$,$(u\bar{u} - d\bar{d})/\sqrt{2}$,$d\bar{u}$; and also isotopic singlets ($I = 0$): $(u\bar{u} + d\bar{d})/\sqrt{2}$. Figure 1.4(a) is an energy-level diagram of the lowest energy isosinglets, including states --- which are interpreted as $s\bar{s}$ states. Figure 1.4(b) is an energy-level diagram of the lowest energy isotriplets. Figure 1.4(c) is an energy-level diagram of the lowest energy K mesons. The K mesons are quark–antiquark systems $u\bar{s}$ and $d\bar{s}$; they are isotopic doublets, as are their antiparticle states $s\bar{u}$ and $s\bar{d}$. Their higher energies relative to the states in Fig. 1.4(b) are largely due to the higher mass of the s over the u and d quarks. The large relative displacement of the 0^+ state is a feature with, as yet, no clear interpretation.

energies 10 GeV and the threshold energy, around 90 GeV, at which the Z boson can be produced.

The e^+e^- annihilation cross-section $\sigma(e^+e^- \to \mu^+\mu^-)$ is comparatively easy to measure, and is easy to calculate in the Weinberg–Salam electroweak theory, which we shall introduce in Chapter 12. At centre of mass energies much below 90 GeV the cross-section is dominated by the electromagnetic process represented by the Feynman diagram of Fig. 1.6. The muon pair are produced 'back-to-back' in the

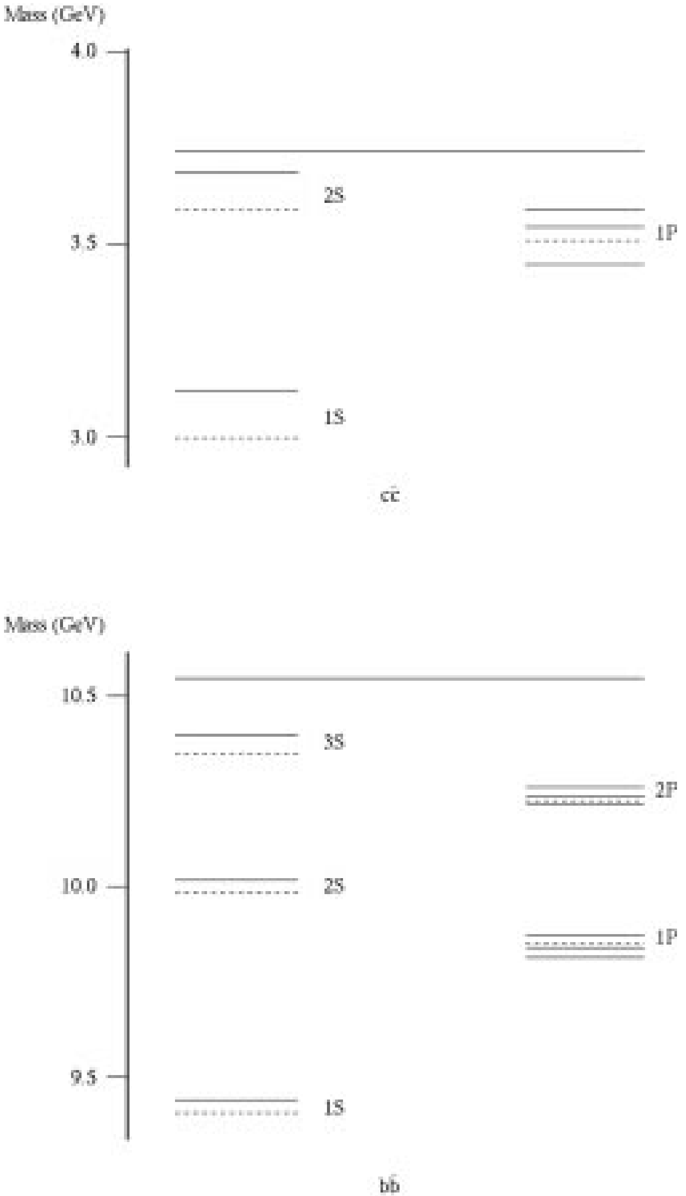

Figure 1.5 Energy-level diagrams for charmonium $c\bar{c}$ and bottomonium $b\bar{b}$ states, below the threshold at which they can decay through the strong interaction to meson pairs (for example $c\bar{c} \to c\bar{u} + u\bar{c}$). States labelled 1S, 2S, 3S have orbital angular momentum $L = 0$ and the 1P, 2P states have $L = 1$. The intrinsic quark spins can couple to $S = 0$ to give states with total angular momentum $J = L$. These states are denoted by ----; experimentally they are difficult to detect. The intrinsic quark spins can also couple to give $S = 1$. States with $S = 1$ are denoted by ——. Spin–orbit coupling splits the P states with $S = 1$ to give rise to states with $J^P = 0^+, 1^+, 2^+$. This spin–orbit splitting is apparent in the figure. All the $S = 1$ states shown have been measured.

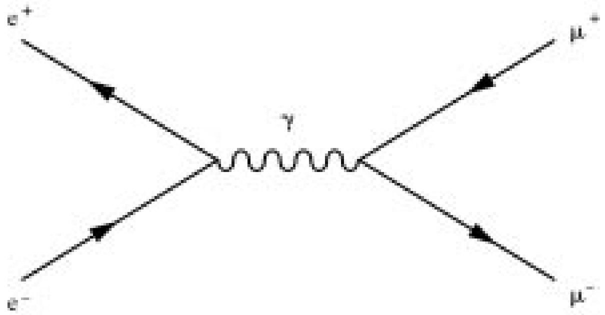

Figure 1.6 The lowest order Feynman diagram (Chapter 8) for electromagnetic $\mu^+\mu^-$ pair production in e^+e^- collisions.

centre of mass system, which for an e^+e^- collider is the laboratory system. To leading order in the fine-structure constant $\alpha = e^2/(4\pi\varepsilon_0\hbar c)$, the differential cross-section for producing muons moving at an angle θ with respect to unpolarised incident beams is

$$\frac{d\sigma}{d\theta} = \frac{\pi\alpha^2}{2s}(1 + \cos^2\theta)\sin\theta \tag{1.1}$$

where s is the square of the centre of mass energy (see Okun, 1982, p. 205). In the derivation of (1.1) the lepton masses are neglected. Integrating with respect to θ, the total cross-section is

$$\sigma = \frac{4\pi\alpha^2}{3s}. \tag{1.2}$$

The quantity $R(E)$ shown in Fig. 1.7 is the ratio

$$R = \frac{\sigma(e^+e^- \to \text{strongly interacting particles})}{\sigma(e^+e^- \to \mu^+\mu^-)}. \tag{1.3}$$

At the lower energies many hadronic states are revealed as resonances, but R seems to become approximately constant, $R \approx 4$, at energies above 10 GeV up to about 40 GeV.

As fundamental particles, quarks have the same electrodynamics as muons, apart from the magnitude of their electric charge. The Feynman diagrams which dominate the numerator of R in this range 10 GeV to 40 GeV are shown in Fig. 1.8. (The top quark has a mass $\sim$ 180 GeV and will not contribute.) For each quark process the formula (1.2) holds, except that e is replaced by the quark's electric charge at the quark vertex, which suggests

$$R = \left(\frac{2}{3}\right)^2 + \left(\frac{1}{3}\right)^2 + \left(\frac{2}{3}\right)^2 + \left(\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^2 = \frac{11}{9}. \tag{1.4}$$

This value is too low, by a factor of about 3.

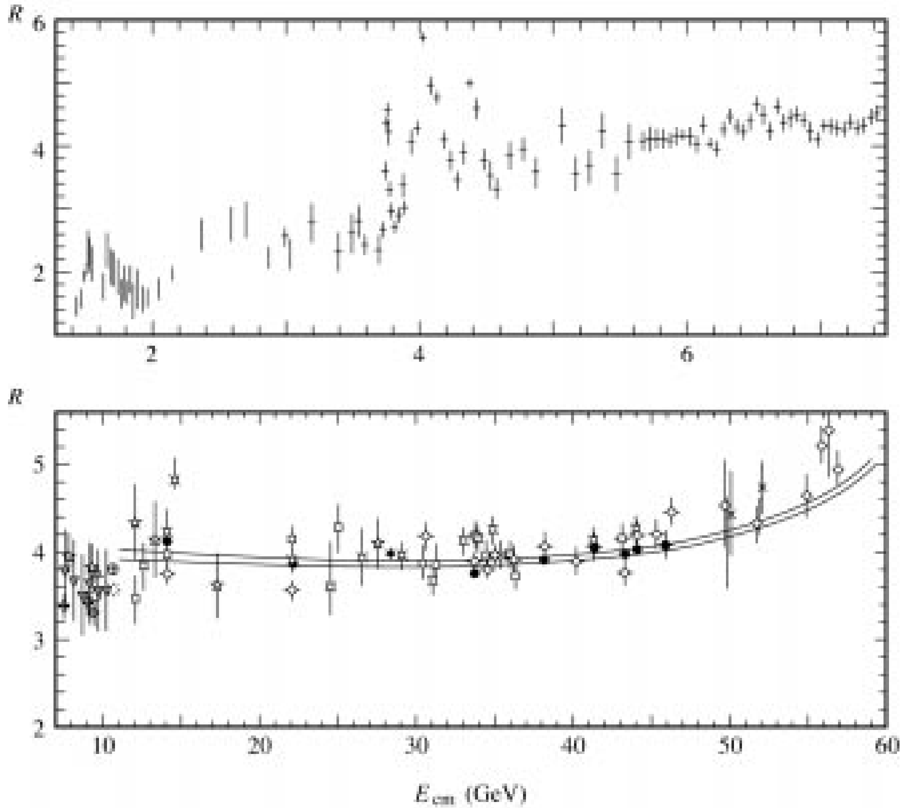

Figure 1.7 Measurements of $R(E)$ from the resonance region 1 GeV $< E <$ 11 GeV into the region 11 GeV $< E <$ 60 GeV which contains no prominent resonances and no quark–antiquark production threshold. For $E >$ 11 GeV two curves are shown of calculations that take account of quark colour and include electroweak corrections and strong interaction (QCD) effects. (Adapted from Particle Data Group (1996).)

In the Standard Model, the discrepancy is resolved by introducing the idea of quark *colour*. A quark not only has a flavour index, u, d, s, c, b, t, but also, for each flavour, a colour index. There are postulated to be three basic states of colour, say red, green and blue (r, g, b). With three quark colour states to each flavour, we have to multiply the R of (1.4) by 3, to obtain

$$R = \frac{11}{3}, \tag{1.5}$$

which is in excellent agreement with the data of Fig. 1.7.

This invention of colour not only solves the problem of R but, most significantly, solves the problem of the symmetry of the baryon states. We have seen (Section 1.5) that in the absence of any new quantum number baryon states are completely

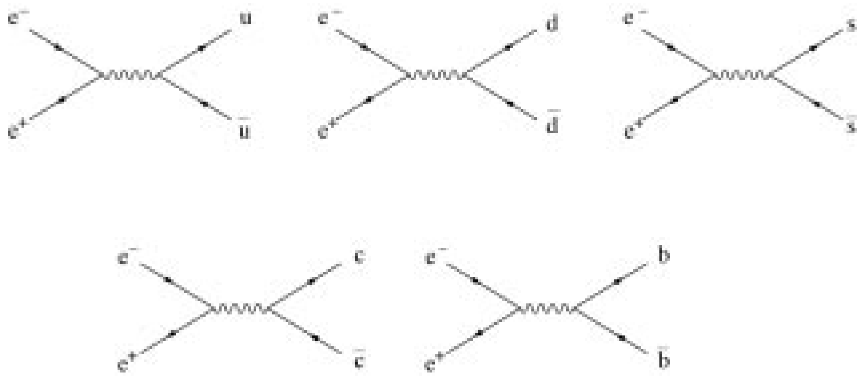

Figure 1.8 The lowest order Feynman diagrams for quark–antiquark pair production in e^+e^- collisions at energies below the Z threshold.

symmetric in the interchange of two quarks. However, if these state functions are multiplied by an antisymmetric colour state function, the overall state becomes antisymmetric, and the Pauli principle is preserved.

Strong support for the mechanism of quark production represented by the Feynman diagrams of Fig. (1.8) is given by other features in the data from e^+e^- colliders. An e^+e^- annihilation at high energies produces many hadrons. These are mostly correlated into two back-to-back *jets*. An example is shown in Fig. 1.9. (The charged particle tracks are curved because of the presence of an external magnetic field: the curvature is related to the particle's momentum.) The direction of a jet may be defined as the direction at the point of production of the total momentum of all the hadrons associated with it. The momenta of two back-to-back jets are equal and opposite. The jet directions may be presumed to be the directions of the initial quark–antiquark pair. This interpretation is corroborated by an examination of the angular distribution of the jet directions of two-jet events from many annihilations, with respect to the e^+e^- beams. The angular distribution is the same as that for muons (equation (1.1)) after allowance has been made for the Z contribution, which becomes significant as the energy for Z production is approached.

The hadron jets result from the original quark and antiquark combining with quark–antiquark pairs generated from the vacuum. The precise details of the processes involved are not yet fully understood.

1.8 Electron scattering from nucleons

There is a clear advantage in using electrons to probe the proton and neutron, since electrons interact with quarks primarily through electromagnetic forces which are well understood; the weak interaction is negligible in the scattering process, except at very high energy and large scattering angle, and the strong interaction is not directly involved.

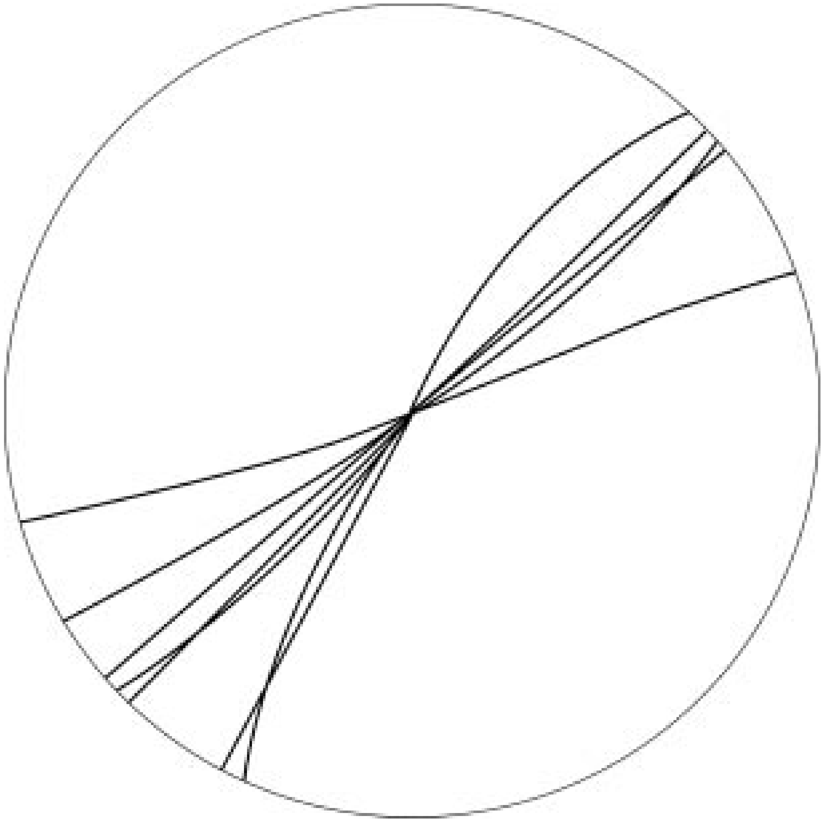

Figure 1.9 An example of an e^+e^- annihilation event that results in two jets of hadrons. The figure shows the projection of the charged particle tracks onto a plan perpendicular to the axis of the e^+e^- beams. This figure was taken from an event in the TASSO detector at PETRA, DESY.

In the 1950s, experiments at Stanford on nucleon targets at rest in the laboratory revealed the electric charge distribution in the proton and (using scattering data from deuterium targets) the neutron. These early experiments were performed at electron energies ≤ 500 MeV (Hofstadter *et al.*, 1958). Scattering at higher energies has thrown more light on the behaviour of quarks in the proton. At these energies inelastic electron scattering, which involves meson production, becomes the dominant mode.

At the electron–proton collider HERA at Hamburg, a beam of 30 GeV electrons meets a beam of 820 GeV protons head on. Many features of the ensuing electron–proton collisions are well described by the *parton model*, which was introduced by Feynman in 1969. In the parton model each proton in the beam is regarded as a system of sub-particles called *partons*. These are quarks, antiquarks and gluons. Quarks and antiquarks are the particles which carry electric charge. The basic idea of the parton model is that at high energy–momentum transfer Q^2, an electron scatters from an effectively free quark or antiquark and the scattering process is

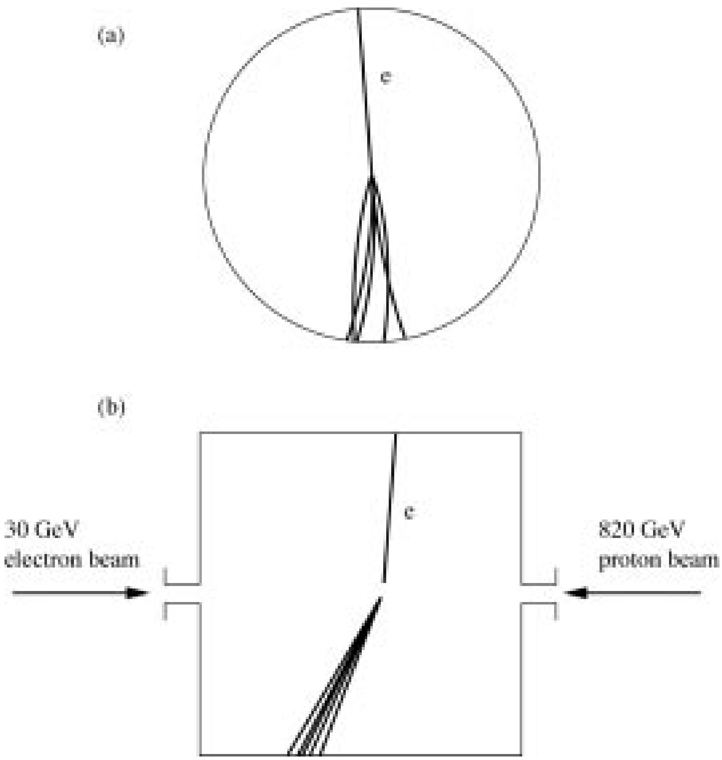

Figure 1.10 This figure illustrating particle tracks is taken from an event in the ZEUS detector at HERA, DESY. Figure 1.10(a) is the event projected onto a plane perpendicular to the axis of the beams. Figure 1.10(b) is the event projected onto a plane passing through the axis of the beams.

A hadron jet has been ejected from the proton by an electron. The track of the recoiling electron is marked e. The initiating beams and the proton remnant jet are confined to the beam pipes and are not detected.

completed before the recoiling quark or antiquark has time to interact with its environment of quarks, antiquarks and gluons. Thus in the calculation of the inclusive cross-section the final hadronic states do not appear.

In the model, at large Q^2 both the electron and the struck quark are deflected through large angles. Figure 1.10 shows an example of an event from the ZEUS detector at HERA. The transverse momentum of the scattered electron is balanced by a jet of hadrons which can be associated with the recoiling quark. Another jet, the 'proton remnant' jet is confined to small angles with respect to the proton beam. Events like these give further strong support to the parton model.

The success of the parton model in interpreting the data gives added support to the concept of quarks. The parton model is not strictly part of our main theme, but in view of its interest, and importance in particle physics, a simple account of the model and its relation to experiment is given in Appendix D.

Table 1.5. *Some particle accelerators*

Machine	Particles collided	Start date
TEVATRON (Fermilab, Batavia)	p: 900 GeV $\bar{p}$: 900 GeV	1987
SLC (SLAC, Stanford)	e^+: 50 GeV e^-: 50 GeV	1989
HERA (DESY, Hamburg)	e: 30 GeV p: 820 GeV	1992
LEP2 (CERN, Geneva)	e^+: 81 GeV e^-: 81 GeV	1996
PEP-II (SLAC, Stanford)	e^-: 9 GeV e^+: 3.1 GeV	1999
LHC (CERN, Geneva)	p: 7 TeV p: 7 TeV	2004

1.9 Particle accelerators

Progress in our understanding of Nature has come through the interplay between theory and experiment. In particle physics, experiment now depends primarily on the great particle accelerators, and ingeneous and complex particle detectors, which have been built, beginning in the early 1930s with the Cockroft–Walton linear accelerator at Cambridge, UK, and Lawrence's cyclotron at Berkeley, USA. The Cambridge machine accelerated protons to 0.7 MeV; the first Berkeley cyclotron accelerated protons to 1.2 MeV. For a time after 1945 important results were obtained using the cosmic radiation as a source of high energy particles, events being detected in photographic emulsion, but in the 1950s new accelerators provided beams of particles of increasingly high energies. Some of the machines now in operation, or planned, are listed in Table 1.5. Detailed parameters of these machines, and of others, may be found in Particle Data Group (1996).

The TEVATRON at Fermilab is where the top quark was recently (1995) observed. The physics of the top quark is as yet little explored. It makes only a brief appearance in our text, though it is an essential part of the pattern of the Standard Model. The upgraded LEP2 at CERN is able to create W^+W^- pairs, and will allow detailed studies of the weak interaction. At Stanford, PEP-II and the associated 'BaBar' ($B\bar{B}$) detector is designed to study charge conjugation, parity (*CP*) violation. The way in which *CP* violation appears in the Standard Model is discussed in Chapter 18.

The most ambitious machine likely to be built in the immediate future is the Large Hadron Collider (LHC) at CERN. It is expected that with this machine it will be possible to observe the Higgs boson, if such a particle exists. The Higgs boson is an essential component of the Standard Model; we introduce it in Chapter 10. It is also widely believed that the physics of Supersymmetry, which perhaps underlies the Standard Model, will become apparent at the energies, up to 14 TeV, which will be available at the LHC.

1.10 Units

In particle physics it is usual to simplify the appearance of equations by using units in which $\hbar = 1$ and $c = 1$. In electromagnetism we set $\varepsilon_0 = 1$ (so that the force between charges q_1 and q_2 is $q_1 q_2/4\pi r^2$), and $\mu_0 = 1$, to give $c^2 = (\mu_0\varepsilon_0)^{-1} = 1$. We shall occasionally reinsert factors of $\hbar$ and c where it may be reassuring or illuminating, or for the purposes of calculation. It is useful to remember that

$$\hbar c \approx 197\ \text{MeV fm}, \qquad e^2/4\pi \approx 1.44\ \text{MeV fm},$$
$$\alpha = e^2/4\pi\hbar c \approx (1/137), \qquad c \approx 3 \times 10^{23}\ \text{fm s}^{-1}.$$

Energies, masses, and momenta are usually quoted in MeV or GeV, and we shall follow this convention.